STUDIES ON MIXING. XXXVI.*

AXIAL HIGH-SPEED ROTARY MIXER AS AN AXIALLY SYMMETRICAL TURBULENT JET**

I. FOŘT[a], M. HRUBÝ[b] and P. MOŠNA[b]

[a] *Department of Chemical Engineering, Institute of Chemical Technology, 16 628 Prague 6,*
[b] *Research Institute of Macromolecular Chemistry, Brno*

Received November 2nd, 1971

Axial high-speed rotary mixer (propeller or blade mixer with inclined plane blades) situated axially in a cylindrical vessel with radial baffles can be considered as an axially symmetrical turbulent jet. Due to its action a free jet forms which is by the bottom and walls of the vessel reversed from downward to upward direction. Radial profile of the mean time velocity in the mentioned free jet may be expressed in a dimensionless form by transformation of the maximum velocity and the radial coordinate where the mean time velocity equals one half of the maximum velocity. These transformed profiles are identical for all sizes of axial mixers at the given arrangement when flow of the charge is turbulent. Intensity of turbulence in this stream is independent both on the radial coordinate and dimensions of the axial mixer as long as the maximum velocity is considered to be the reference for the mean time velocity. Heat transfer intensity between the vessel bottom and the mixed charge can be determined on the basis of relations for heat transfer between the flat plate and turbulent stream which is flowing around it as long as the maximum velocity in the free stream before its interaction with the solid wall or the vessel bottom is considered to be the characteristic velocity. The obtained results are valid in the range of relative mixer sizes $d/D \in \langle 1/5;\ 1/3 \rangle$ and hydrodynamic conditions in the mixed system $\mathrm{Re} \in \langle 1{\cdot}0 \cdot 10^4;\ 2{\cdot}0 \cdot 10^5 \rangle$.

Turbulent flow of a Newtonian charge mixed by an axial high-speed rotary mixer (propeller or blade mixer with inclined plane blades) in a cylindrical vessel with radial baffles has not yet been described, *i.e.* no model was proposed which would include both the basical components of turbulent motion of fluids – the mean and fluctuation components. On the contrary, the free or impinging axially symmetrical turbulent flow resulting from jets of various shapes has already been described in its variety in a satisfactory way so that the mass and heat transfer intensities between such stream and the surrounding environment (or in the stream) may be determined with a sufficient accuracy. In this paper some kinematic properties of the axially symmetrical stream formed by a rotating axial high-speed mixer are discussed so that it is possible to find congruent features in hydrodynamic behaviour of this stream and the

* Part XXXV: This Journal *37*, 2371 (1972).

** Presented at the Meeting "Mischvorgänge" Würzburg 1973.

classical turbulent jet. The determined results are further compared with the procedure for determination of the heat transfer intensity between the flat wall and free jet impacting on this wall when the source of the considered jet is the axial high-speed rotary mixer.

The velocity field of liquid in turbulent flow mixed by an axial high-speed rotary mixer in a system with radial baffles has already been studied. Majority of these studies are of an experimental character[1–13], but theoretical considerations concerning the velocity field at the exit from blades of an axial high-speed mixer[4,5,13,15] have also been made as well as studies on the field of stream-lines in the charge[16,17]. From majority of these studies results that in the liquid stream leaving the blades of a rotating mixer and in the same stream after reversal of its direction due to the vessel bottom and walls there exists a considerable radial profile of the mean time velocity. Further, in the horizontal plane in which the lower edges of mixer blades are situated radius of the downward stream is greater than the mixer radius due to the momentum transfer between the liquid present in the considered body formed by a rotating mixer and the surrounding media. Intensity of turbulence in the flow streaming from the blades of a rotating axial mixer was found to be considerable but none of theoretical methods for determination of the velocity field in the considered system either enabled to include this quantity into the final relations or at least to estimate it. Convective flow of the mixed charge as the source of transfer of properties in the system does not include the effect of all kinematic quantities which has been already confirmed by studies of heat transfer between the vessel bottom and the mixed charge[18] or at suspension of the solid phase[19] in a system with an axial mixer. This is because the convective flow of the charge cannot be considered separately but the convective transport of a whole spectrum of turbulent eddies originating at separation of the boundary layer and wake behind the blades of a rotating mixer[20] must be taken into account. This model has been found suitable already in studies on flow at the exit from blades of a radial mixer (*e.g.* turbine mixer) and therefore it was possible to solve the considered flow problem with regard to the assumptions related to theories of free or impinging jets[21] as one of the given cases.

The high-speed axial rotary mixer may be considered to be a source of convective transfer of intensive turbulent flow. The velocity field in the formed stream will consists of two – as concerns their size-quite comparable components – the mean time and fluctuation components as in the case of free or impinging jets. Flow studies concerning the mixed charge based on the jet theories can thus give better *i.e.* more complete description of the complex velocity field in the considered system.

THEORETICAL

Let us consider a mixed system formed by a Newtonian charge in a cylindrical vessel with flat bottom and with four radial baffles. The lower edge of baffles is in contact with the bottom and the upper edge is above the liquid surface. An axial high-speed rotary mixer rotates in the vessel oriented so as to force the liquid toward the vessel bottom. The mixed system defined in this way is considered to be axially symmetrical. In this system a region below the horizontal plane of axial symmetry of the mixer is chosen (Fig. 1) and in the axial plane of the system the velocity field *i.e.* the field of streamlines is analysed: By rotation of the mixer blades forms an axially symmetrical free jet *I* where the radius r_∞ is greater than the mixer radius $d/2$ due to the effect of radial momentum transfer between the liquid present in the considered body formed by the rotating mixer and the surrounding media. Because of the vessel bottom the free jet begins to reverse its direction just behind the horizontal plane in which the lower edges of the mixer blades are situated and thus it changes its character into the so called impact jet *II* with a continuous change of its direction. Simultaneously in vicinity of the vessel axis forms a conical region in which the streaming is practically suppressed and which is called the "dead region". The cone bottom has a radius r_{sp}. The stream after its reversal at the bottom is called the radial wall jet *III*. Its velocity profile is affected by the profile of the preceding free jet and by the presence of solid walls as boundaries. The wall jet thus formed becomes, due to the action of the wall, an impact jet *IV* again when a continuous change of the flow direction takes place. If reversal of this flow takes place, character of the stream is again given by the solid walls forming its boundaries and by the radial velocity profile. This profile depends on the preceding history of the stream especially on the shape of velocity profile at the exit from the blades of the rotating mixer. Thus another, this time the axial wall jet *V* forms.

On basis of studies of the velocity field in the liquid stream below a rotating axial high-speed rotary mixer[10,11] the following equations may be written (see also Fig. 1) for the radial mean time velocity and intensity of turbulence

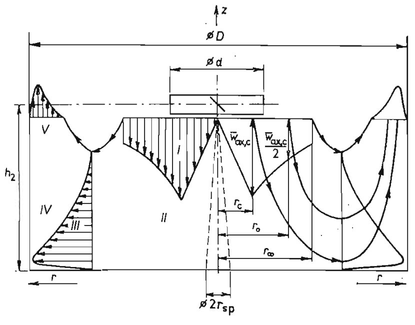

FIG. 1

Flow Model Below the Horizontal Plane of Symmetry of Axial High-Speed Rotary Mixer in Cylindrical Vessel with Radial Baffles

I Free axially symmetrical jet, *II* axially symmetrical impact jet at the bottom, *III* axially symmetrical radial wall jet at the bottom, *IV* axially symmetrical impact jet at the wall, *V* axially symmetrical axial wall jet at the wall.

I. $r \in (0; r_c\rangle$:

$$\bar{w}_{ax} = 2\pi nkr\,, \quad (\overline{w_{ax}'^2})^{1/2}/\bar{w}_{ax} = c/r\,, \tag{1a,b}$$

II. $r \in \langle r_c; r_\infty\rangle$:

$$\bar{w}_{ax} = 2\pi nC/r, \quad (\overline{w_{ax}'^2})^{1/2}/\bar{w}_{ax} = Kr\,. \tag{2a,b}$$

In relations (*1a*)–(*2b*), constants c, C and K are dependent on the size and type of the axial rotary mixer and are obtained experimentally. Quantity k in Eq. (*1a*) is dependent only on the type of the axial mixer. For blade mixer with plane blades inclined to the horizontal plane under the angle γ the mentioned quantity holds[4]

$$k = \operatorname{cotg}\gamma \tag{3}$$

which for propeller mixer with a constant slope of the helics $s = Nd$ equals[15]

$$k = N/\pi\,. \tag{4}$$

But here, because of the buoyancy effect of the areodynamic shape of blades, Eqs (*1a*) and (*2a*) for the propeller mixer[15] become

$$\bar{w}_{ax} = 2\pi nkr + 2\pi n\Theta\,, \quad \bar{w}_{ax} = (2\pi nC/r) + 2\pi n\Theta\,. \tag{1c, 2c}$$

As relations (*1c*) and (*2c*) may be transformed to the form of Eqs (*1a*) and (*2a*) by introducing the quantity

$$\bar{w}_{ax,norm} = \bar{w}_{ax} - 2\pi n\Theta\,, \tag{5}$$

only these equations are further discussed. On their basis for the maximum velocity $\bar{w}_{ax,c}$ and for its coordinate r_c the following relations are obtained

$$\bar{w}_{ax,c} = 2\pi n(kC)^{1/2}\,, \quad r_c = (C/k)^{1/2}\,, \tag{6a,b}$$

so that the coordinate of location r_0, in which the mean time velocity is reaching the value $\bar{w}_{ax,c}/2$ equals

$$r_0 = 2r_c = 2(C/k)^{1/2}\,. \tag{7}$$

If relations (*1a*) and (*2a*) are transformed to a dimensionless form, the following relations are obtained

I. $r \in \langle 0; r_c\rangle$:

$$\bar{w}_{ax}/\bar{w}_{ax,c} = 2r/r_0\,, \tag{8a}$$

II. $r \in \langle r_c; r_\infty\rangle$:

$$\bar{w}_{ax}/\bar{w}_{ax,c} = r_0/2r\,. \tag{8b}$$

Relations (*8a*) and (*8b*) are already independent on the size and type of the axial rotary mixer. But at the same time they satisfy the conditions that at the origin they are equal to zero, for $r = r_0$ are reaching maximum values (are equal to one) and for $r \to r_0$ are equal to $^1/_2$. From relations (*1b*) and (*2b*) for the root mean square values of the fluctuation velocity component, by use of Eqs (*1a*) and (*2a*) the following relations are obtained

I. $r \in (0; r_c\rangle$:

$$\left(\overline{w'^2_{ax}}\right)^{1/2} = 2\pi nkc\,, \tag{9a}$$

II. $r \in \langle r_c; r_\infty \rangle$:

$$\left(\overline{w'^2_{ax}}\right)^{1/2} = 2\pi nKC\,. \tag{9b}$$

With respect to relation (*6b*) and to equality

$$K = c/r_c^2\,, \tag{10}$$

Eq. (*9b*) may be arranged to the form

II. $r \in \langle r_c, r_\infty \rangle$:

$$\left(\overline{w'^2_{ax}}\right)^{1/2} = 2\pi nkc\,. \tag{11}$$

Thus the root mean square of the fluctuation velocity is for the given mixer type independent on the position along the radial axis r and is dependent only on the frequency of revolution n of the mixer. For the relative intensity of turbulence related to quantity $\bar{w}_{ax,c}$ simultaneously holds

$$\left(\overline{w'^2_{ax}}\right)^{1/2}/\bar{w}_{ax,c} = c(k/C)^{1/2}\,, \quad [r \in (0; r_\infty\rangle]\,, \tag{12}$$

which is a quantity practically independent on the size of the given type of axial rotary mixer as results from experiments[11] (see further). The free axially symmetrical jet leaving the blades of the axial high-speed rotary mixer may thus be expressed by a dimensionless radial profile of the mean time velocity in agreement with the similarity of velocity profiles in the mixing zone of classical free jets[21] and by the constant relative turbulence intensity $\left(\overline{w'^2_{ax}}\right)^{1/2}/\bar{w}_{ax,c}$.

The impact jet *II* originated by interaction of the above described free jet and the vessel bottom is by its structure similar to the free jet which results from the mixer rotation. But the radial profile of the mean time velocity already includes not only the axial component $\bar{w}_{ax}$ but due to the action of the bottom the radial component $\bar{w}_{rad}$ as well. The profile $\bar{w}_{ax} = \bar{w}_{ax}(r)$ in the whole region of the impact jet *i.e.* along the axial coordinate z (see Fig. 1) is similar to the profile given by relations (*1a*) and (*2a*) with the hyperbolical shape of the descending part of the profile distorted by radial pulsations to the linear shape[11,12]. Slopes in both profiles are decreasing both with the successive reversal of the stream and toward the bottom (*i.e.* in the direction of decreasing values of coordinate z) while the beginning of the increasing part of the profile moves toward the wall as in vicinity of the system axis the "dead region" becomes greater. The average value of the turbulence intensity $(\overline{w'^2})^{1/2}/\overline{w}$ across the whole cross sectional area of flow in the considered region is practically constant[11] and equal to 1/3 regardless of the relative size of the mixer. This means, that this value is practically equal to that in the stream just leaving the blades of the rotating mixer *I*. Of course, simultaneously by the action of the vessel bottom in this stream, tangential component of the mean velocity, which results from rotation of the mixer[7] decreases. The decrease of this quantity toward the vessel bottom results not only from the action of the vessel bottom but mainly from the action of radial baffles.

Heat Transfer Between the Mixed Charge and the Bottom

Flow of the charge mixed by an axial high-speed rotary mixer in agreement with the presented model in vicinity of the bottom is described by the mechanism of the impact jet *II* and by the mechanism of the wall jet[22,23] *III*. The region of the bottom must be divided into the part in vicinity of the stagnation point (coordinate r_{sp}) where the character of flow is not yet ensuring a purely turbulent transfer and further farther from the stagnation point where already a purely turbulent transfer takes place. At the same time in part of the bottom with the radial coordinate less than r_{sp} natural convection becomes expressive together with eventual laminar transfer mechanism. In the region of bottom where the heat transfer mechanism can be considered fully turbulent may be assumed that the axial velocity profile in vicinity of the wall is fully developed. From results of the study on wall jets[21] the "one seventh" law was proved to be valid for this region, while the velocity profiles are similar with respect to transformations (*8a*) and (*8b*). With regard to the preserved turbulent structure of the stream between the rotating axial mixer and the bottom, this structure may be considered as well to be preserved in the radial wall jet the kinematic expression of which then corresponds to relations (*8a*) and (*8b*) and (*12*) in the bulk stream leaving the blades of the rotating mixer. In this case for heat transfer in the turbulent boundary layer[24] between the bottom and the charge in the point r, under assumption of constant temperature of both media, may be written the relation

$$\mathrm{Nu}_r(r) \equiv [\alpha(r)\,(r - r_{sp})]/\lambda = 0{\cdot}0292\ \mathrm{Re}_r^{0\cdot8}\mathrm{Pr}^m\,, \qquad (13)$$

where the Reynolds number Re_r is defined by

$$\mathrm{Re}_r \equiv [\overline{w}_{ax,c}(r - r_{sp})]/\nu\,, \qquad (14)$$

i.e. as a characteristic velocity is considered the maximum velocity $\overline{w}_{ax,c}$ of the free jet which with respect to Eq. (*12*) is also the quantity representing the intensity of turbulence in the considered stream. This is in an agreement with the above made assumption of preservation of the turbulent structure of streams which are mutually interconnected. The Reynolds number for flow along the bottom as a flat plate may be arranged by use of transformations

$$\mathrm{Re}_r = [(r - r_{sp})/r_0]\,(\bar{w}_{ax,c}\,r_0)/\nu\,, \qquad (15)$$

$$r_0 = K_1 d/2\,, \quad \bar{w}_{ax,c} = K_2 \pi d n\,, \quad r - r_{sp} = K_3 D\,. \qquad (16-18)$$

By substituting into Eq. (*13*) we obtain

$$\frac{\alpha(r)\,D}{\lambda} \equiv \mathrm{Nu}_M(r) = \frac{0{\cdot}0292(\pi K_2)^{0{\cdot}8}}{K_3^{0.2}}\,(D/d)^{0{\cdot}8}\mathrm{Re}_M^{0{\cdot}8}\mathrm{Pr}^m\,, \qquad (19)$$

where the mixing Reynolds number Re_M is defined by

$$\mathrm{Re}_M \equiv nd^2/\nu\,. \qquad (20)$$

Value of the exponent m in the Prandtl number with regard to results of experiments on liquid mixing at turbulent flow regime may be chosen[25] $m = 0{\cdot}45$.

From the known kinematic characteristics of the free jet at the exit from blades of an axial high-speed rotary mixer, the radial profile of the local heat transfer coefficient $\alpha = \alpha(r)$ between the bottom and the charge may be determined in the region of fully developed turbulent flow at the bottom (*i.e.* not only in vicinity of the point $r = r_{sp}$).

EXPERIMENTAL

The experiments were carried out in a cylindrical vessel with flat bottom with four radial baffles situated at the wall. The baffles were $0{\cdot}1D$ wide. The vessel was filled with a homogeneous Newtonian charge (distilled water or aqueous solution of glycerol) so that the distance of its surface from the vessel bottom H, (when at rest) was equal to the inside diameter of the vessel D. A six blade mixer with inclined ($\gamma = 45°$) plane blades was rotating in the vessel axis. This mixer was described in previous papers of this series[13,26]. Direction of the mixer rotation was such as to pump the liquid toward the vessel bottom. The distance of the horizontal plane of mixer symmetry above the bottom was kept constant and equal to $h_2 = 0{\cdot}25D$. In the described mixing system two series of measurements were made: *1*. In the vessel with the inside diameter $D = 0{\cdot}290$ m were determined kinematic flow characteristics of the mixed charge in the region below the horizontal plane of mixer symmetry. The measurements were made by use of the oriented three-hole Pitot tube[7], three oriented Pitot tubes[12] and finally by the photographic method of traces[4]. As independent variables were set in the experiments frequency revolution n of the mixer used and the relative mixer and vessel sizes d/D. Resulting were the radial profiles of the mean time velocity[10] and intensity[11] of turbulence in the stream leaving the blades of a rotating mixer (region *I* in Fig. 1) and further the radial profiles of the mentioned quantities at the vessel bottom[12]. From the results of measurements were obtained also informations on the field of the mean time velocity in the space of the jet impacting on the bottom (region *II*)[7] and also in the space of the axial wall jet at the vessel wall (region *V*)[7]. Measurements were made in turbulent flow regime of the mixed charge *i.e.* at $\mathrm{Re}_M > 1{\cdot}0 \cdot 10^4$. For a detailed description of the used measuring procedures as well as for evaluation of their accuracy reference is made to the cited papers of this series.

2. In a vessel with the inside diameter $D = 0{\cdot}240$ m at constant density of the heat flux $\dot{q}_b$ were determined the temperatures in the respective points of the vessel bottom[18,27] by use of thermocouples situated in the vessel bottom. Location of measuring thermocouples in the wall of the bottom was chosen with respect to the requirement of fully developed turbulent flow in the region above these measuring elements. The mentioned thermocouples were located on the radial ray situated in the axis of symmetry between two adjacent baffles in the relative distance $r/D = 0{\cdot}375$ from the axis of system symmetry. There are four such radial rays on the vessel bottom at the given geometrical arrangement. For increasing the accuracy of the measured voltage taken by the thermocouple was located on each ray in the given point one measuring element. So it was possible to obtain the voltage of four thermocouples connected in series and from it the mean temperature in the given point of the bottom was calculated. For verification of considerations made concerning the effect of hydrodynamic conditions on heat transfer intensity between the vessel bottom and the mixed charge other four thermocouples were situated on identical radial rays in vicinity of the coordinare r_{sp} at the relative distance $r/D = 0{\cdot}100$ from the axis of system symmetry. All experimental results were then evaluated on basis of the known temperature $t_w(r)$ of the wall of the bottom in the point r, temperature of the charge t, density of heat flux between the bottom and the charge $\dot{q}_b$ and thermal conductivity of the charge λ into the dimensionless form according to relation

$$\mathrm{Nu}_M(r) = \dot{q}_b D/\lambda[t_w(r) - t]\,, \qquad (21)$$

so that a comparison with considerations resulting from the hydrodynamic behaviour of the studied system was possible.

RESULTS AND DISCUSSION

Axially Symmetrical Free Jet and Stream Impaction on the Bottom

For the six-blade mixer with inclined ($\gamma = 45°$) plane blades are in Table I given parameters of radial profiles of quantity $\bar{w}_{ax}$ and $(\overline{w'^2_{ax}})^{1/2}/\bar{w}_{ax,c}$ as they result from the experiments[10,11]. The value of constant k for this type of mixer is on basis of Eq (*3*) equal to one. But it is necessary to mention that in the considered cross sectional area for flow (*i.e.* for horizontal plane in which the lower edges of mixer blades are situated) there exists beside the tangential flow component only the axial velocity component. In the axial direction toward the bottom is appearing also the radial centrifugal component both in the outward (at the boundary of the stream) and in inward (inside the stream) direction.

Validity of our assumptions concerning the similarity of the corresponding radial profiles results from Table I, *i.e.* the proposed transformation methods are independent on the mixer size. Though the mixer diameter has increased by more than 66% the quantity $2r_0/d$, characterizing the dimensionless radial distance has not changed its value by more than 6·5% with the quantity $\bar{w}_{ax}$ equal to one half of the value of quantity $\bar{w}_{ax,c}$. In a similar way also behaves the quantity $\bar{w}_{ax}/\pi dn$. The dimensionless relative intensity of turbulence $(\overline{w'^2_{ax}})^{1/2}/\bar{w}_{ax,c}$ also does not change by more than 15%. Greater deviation is caused by a relatively large value of standard deviation of this quantity, with its relative value σ_{rel} also given in Table I. For all the discussed quantities the deviation is systematic for the mixer with the relative size $d/D = 1/4$. This fact can be explained by different relative size of the hub d_h/d which is greater than with mixers of relative sizes $d/D = 1/3$ and $1/5$, *i.e.* geometrical similarity is not fulfilled in this case.

Heat Transfer between the Vessel Bottom and Mixed Charge

From measurements of the heat transfer intensity in the given point r of the bottom were calculated values of the local Nusselt mixing number $\mathrm{Nu_M}(r)$ (Eq. (*21*)). For the given material properties of the charge were also calculated values of the Prandtl number $\mathrm{Pr} = \nu/a$ and of the mixing Reynolds number $\mathrm{Re_M}$ (Eq. (*20*)). By the method of least squares were calculated the parameters a and A of the exponential dependence

$$\mathrm{Nu_M}(r) = A(r)\,\mathrm{Re_M^{a(r)}}\mathrm{Pr}^{0.45}\,, \quad [d/D = \text{const.}\,; \quad r/D = \text{const.}]\,. \qquad (22)$$

TABLE I

Parameters of Radial Profiles in Mean Time and Fluctuation Velocities in Stream Leaving the Blades of Rotating Axial High-Speed Mixer (six-blade mixer with inclined plane blades ($\gamma = 45°$), $h_2/D = 0.25$, $\mathrm{Re_M} > 1.0 \cdot 10^4$)

d/D	$d \cdot 10^3$ m	$(C/\pi dn) \cdot 10^3$ m	$r_c \cdot 10^3$ m	$\bar{w}_{ax,c}/\pi\, dn$	$r_0 \cdot 10^3$ m	$2r_0/d$	$(\overline{w'^2_{ax}})^{1/2}/\bar{w}_{ax,c}$	σ_{rel}
1/5	58·0	16·2	21·8	0·747	43·6	1·50	0·152	25
1/4	72·5	17·8	25·6	0·702	51·2	1·41	0·132	22
1/3	96·7	26·2	35·6	0·738	71·0	1·47	0·152	19

TABLE II

Parameters of Exponential Dependence of Nusselt and Reynolds Numbers (six-blade mixer with inclined plane blades ($\gamma = 45°$), $h_2/D = 1/4$, $r/D = 0.375$, $\mathrm{Re_M} > 8.0 \cdot 10^3$, $\mathrm{Pr} = 5.70$)

d/D	A	a	R
1/5	0·131	0·85	0·998
1/4	0·120	0·85	0·998
1/3	0·0657	0·90	0·990

In Table II are given parameters of the given exponential dependence as well as estimates of the corelation coefficient R for each calculated regression dependence. From results given in Table II is obvious, that the value of exponent a is constant for smaller mixers while the value of constant A is increasing with increasing value of the ratio d/D. For a mixer of relative size $d/D = 1/3$, the value of exponent a slightly differs which presents comparison of constants A for this and other mixers.

It was possible to evaluate the relation of dimensionless numbers for the heat transfer intensity (*19*) on basis of the known parameters of radial profiles of the mean time velocity in the liquid stream leaving the blades of a rotating mixer and from the known quantity r (radial coordinate on the bottom where the value of $\mathrm{Nu_M}(r)$ number is determined) and r_{sp}. The quantity r_{sp} was determined from the numerically calculated field of streamlines for the given conditions in the mixed system[17]. It is given in Table III together with the ratio $\mathrm{Nu_M}(r)/\mathrm{Re_M^{0 \cdot 8}Pr^{0 \cdot 45}}$ which is in a dimensionless form characterizing the geometric and kinematic conditions in the free jet and in the radial wall jet at the bottom. The obtained result which is based on the purelly hydrodynamic considerations may be compared with the results of directed measurements of the heat transfer intensity at the vessel bottom in the considered point r. This comparison is made in Table IV where in the range $\mathrm{Re_M} \in \langle 1 \cdot 0 \,.\, 10^4; 2 \cdot 0 \,.\, 10^5 \rangle$ are compared quantities $\mathrm{Nu_M}(r)/\mathrm{Pr^{0 \cdot 45}}$ which were obtained from Eq. (*19*) and from Eq. (*22*). (The value calculated from relation (*22*) is denoted $\mathrm{Nu_{Mexp}}(r)/\mathrm{Pr^{0 \cdot 45}}$). On basis of data given in Table IV deviation of the experimental and calculated values of quantity $\mathrm{Nu_M}(r)/\mathrm{Pr^{0 \cdot 45}}$ is less than 20% for the range $\mathrm{Re_M} \in \langle 1 \cdot 0 \,.\, 10^4; 2 \cdot 0 \,.\, 10^5 \rangle$ with the exception of one case. Greater deviation of the theoretical dependence appears only for the mixer of relative size $d/D = 1/3$ as long as $\mathrm{Re} > 1 \cdot 5 \,.\, 10^5$. This deviation as well as the greater value of exponent a in the regression relation (*22*) confirms the possibility of existence of a different flow mechanism both in the impact and wall jets at the bottom in the system with a mixer of this size so that the effect of tangential velocity component is considerable. It seems that this component is not hindered by the presence of the bottom and radial baffles as it is usual

TABLE III

Heat Transfer Intensity at Vessel Bottom (six-blade mixer with inclined plane blades ($\gamma = 45°$), $\mathrm{Re_M} > 1 \cdot 0 \,.\, 10^4$; $r/D = 0 \cdot 375$; $h_2/D = 0 \cdot 25$)

d/D	r_{sp}/D	$(r - r_{sp})/D$	$\mathrm{Nu_M}(r)/(\mathrm{Re_M^{0 \cdot 8}Pr^{0 \cdot 45}})$
1/5	0·030	0·345	0·259
1/4	0·045	0·330	0·207
1/3	0·075	0·300	0·175

with other two mixers of different sizes. In analyzing the flow mechanism of the charge above the bottom in vicinity of point r_{sp} the flow was considered on basis of the results of measurements of the velocity field in the impact jet on a flat plate[22,23], as a flow laminar or transition regime. These considerations were confirmed by the value of exponent a for the dimensionless equation (*22*) which was determined by measuring the local heat transfer coefficient in vicinity of the mentioned point. From results of measurements made on the bottom in the point $r/D = 0{\cdot}100$ the value of exponent with Re_M equals $a = 1/4$. In this point the effect of natural convection is still considerable and the proposed heat transfer model is not suitable here. From Table IV further follows that the significant effect of the quantity d/D on value of $\mathrm{Nu}_M(r)$ appears to be justified which is in agreement with the results of experimental studies of the intensity of convective flow at the vessel bottom in the considered system[8] as well as with results of studies of heat transfer rate between the whole bottom and the mixed charge[18]. With the results of the last cited paper are also in agreement values of exponent a with the Re_M (and the corresponding exponent in relation (*19*) as well) which corresponds to the dissipative transfer model of properties at the bottom when the axial high-speed rotary mixer is used in a vessel with radial baffles when the flow regime is turbulent. The transfer mechanism is controlled not only by the chaotic turbulent motion of eddies but, in agreement with the conception of the free and wall jets, by their convective transfer. The proposed model makes it also possible to understand the transfer mechanism in the axial wall jet at the wall (at least in the space below the horizontal plane of mixer symmetry). With respect to additional turbulization of the stream resulting from its reversal in the corner at the wall it is possible to consider in this jet as the characteristic velocity in the dimensionless relation for the heat transfer coefficient in turbulent flow along the smooth plate (*13*) again the quantity $\bar{w}_{ax,c}$. Anyway, the radial profile

TABLE IV

Comparison of Experimental Heat Transfer Intensities at Vessel Bottom with Model Results ($r/D = 0{\cdot}375$; $\mathrm{Re}_M > 1{\cdot}0 \,.\, 10^4$)

Re_M	$d/D = 1/5$		$d/D = 1/4$		$d/D = 1/3$	
	$\frac{\mathrm{Nu}_{M\,exp}}{\mathrm{Pr}^{0.45}}$	$\frac{\mathrm{Nu}_M}{\mathrm{Pr}^{0.45}}$	$\frac{\mathrm{Nu}_{M\,exp}}{\mathrm{Pr}^{0.45}}$	$\frac{\mathrm{Nu}_M}{\mathrm{Pr}^{0.45}}$	$\frac{\mathrm{Nu}_{M\,exp}}{\mathrm{Pr}^{0.45}}$	$\frac{\mathrm{Nu}_M}{\mathrm{Pr}^{0.45}}$
$1{\cdot}0 \,.\, 10^4$	335	400	305	335	260	280
$5{\cdot}0 \,.\, 10^4$	1 310	1 490	1 200	1 190	1 120	1 010
$1{\cdot}0 \,.\, 10^5$	2 330	2 590	2 140	2 070	2 080	1 750
$1{\cdot}5 \,.\, 10^5$	3 300	3 580	3 030	2 860	3 000	2 420
$2{\cdot}0 \,.\, 10^5$	4 240	4 460	3 890	3 560	3 870	3 000

of the mean time velocity plotted in Fig. 1 corresponds to the results of experiments made in the studied system[7,9,10]. In studying the heat transfer intensity between the wall and the mixed charge for the part of the wall situated below the horizontal plane of symmetry of the mixer the relations for heat transfer in the turbulent boundary layer at the smooth plane may be used. Origin of the boundary layer must be situated in this case in the point r_{sp} and, because of intensive turbulence of the stream outside the boundary layer, as the characteristic velocity must be considered the maximum velocity $\bar{w}_{ax,c}$ in the axially symmetrical free jet. These results are correcting the conclusions made in one of the previous papers of this series[9] but they are in agreement with the results of experimental determination of the heat transfer intensity between the vessel wall and the mixed charge. Thus the model of flow of the mixed charge in vicinity of the bottom where the liquid stream is assumed a sequence of free, impact and wall jets seems to be adequate and it is possible by its use to explain succesfully the mechanism of heat transfer taking place in this system.

LIST OF SYMBOLS

A	constant in Eq. (*22*)
a	exponent in Eq. (*22*)
a	thermal conductivity of the mixed charge $(m^2 s^{-1})$
C	constant in Eq. (*2a*) (m^2)
c	constant in Eq. (*1b*) (m)
D	inside vessel diameter (m)
d	mixer diameter (m)
d_h	diameter of mixer hub (m)
H	height of liquid surface above the bottom when at rest (m)
h_2	height of the horizontal plane of mixer symmetry above the vessel bottom (m)
K	constant in Eq. (*2b*) (m^{-1})
K_1, K_2, K_3	constants defined by relations (*16*)—(*18*)
k	constant defined by relation (*3*)
m	exponent in Eq. (*19*)
N	quantity defined by relation (*4*)
n	frequency of mixer revolution (s^{-1})
$\dot{q}_b$	density of heat flux at vessel bottom $(kcal\ m^{-2} s^{-1})$
R	correlation coefficient
r	radial distance (coordinate) from the axis of symmetry of the mixed system (m)
r_{sp}	radial coordinate of the stagnation point on bottom (m)
r_{∞}	radius of the free, axially symmetrical jet (m)
s	slope of the propeller mixer (m)
t	temperature (deg)
$\bar{w}$	local mean time velocity (ms^{-1})
w'	fluctuation component of local velocity (ms^{-1})
z	axial distance (coordinate) from vessel bottom (m)
α	heat transfer coefficient $(kcal\ m^{-2} s^{-1} deg^{-1})$
γ	angle of blade inclination from horizontal plane (deg)
Θ	liquid circulation resulting from the lift effect of propeller blades (m)

λ thermal conductivity of mixed charge (kcal m^{-1} s^{-1} deg^{-1})
ν kinematic viscosity of mixed charge (m^2 s^{-1})
σ_{rel} relative deviation (%)
$Nu_M \equiv \alpha D/\lambda$ Nusselt mixing number
$Nu_r \equiv \alpha(r - r_{sp})/\lambda$ Nusselt number
$Pr \equiv \nu/a$ Prandtl number
$Re_M \equiv nd^2/\nu$ Reynolds mixing number
$Re_r \equiv \bar{w}_{ax,c}(r - r_{sp})/\nu$ Reynolds number

Subscripts

c related to the maximum velocity in the free, axially symmetrical jet
norm quantity transformed by addition (subtraction) of constant
o related to one half of maximum velocity in the free jet

REFERENCES

1. Nagata S., Yamamoto K., Hashimoto K., Naruse Y.: Mem. Fac. Eng., Kyoto Univ. *31*, 260 (1959).
2. Porcelli J. V., Marr G. R.: Ind. Eng. Chem. Fund. *1*, 172 (1962).
3. Wolf D., Manning F. S.: Can. J. Chem. Eng. *44*, 137 (1966).
4. Fořt I.: This Journal *32*, 3520 (1967).
5. Cooper R., Wolf D.: Can. J. Chem. Eng. *46*, 94 (1968).
6. Marr G. R.: *Thesis*. Princeton University, New Jersey 1959.
7. Fořt I., Podivínská J., Baloun R.: This Journal *34*, 959 (1969).
8. Fořt I., Eslamy M., Košina M.: This Journal *34*, 3673 (1969).
9. Fořt I., Vlček J., Cink M., Hrubý M.: This Journal *36*, 1546 (1971).
10. Fořt I., Neugebauer R., Pastyříková H.: This Journal *36*, 1769 (1971).
11. Fořt I.: This Journal *36*, 2914 (1971).
12. Kudrna V., Fořt I., Eslamy M., Cvilink J., Drbohlav J.: This Journal *37*, 241 (1972).
13. Fořt I., Sedláková V.: This Journal *33*, 836 (1968).
14. Weber A. P.: Chem. Eng. *70*, No. 18, 91 (1963).
15. Fořt I.: This Journal *34*, 1094 (1969).
16. Muldoon J. F.: *Thesis*. Columbia University, New York 1968.
17. Fořt I., Gračková Z., Koza V.: This Journal *37*, 2371 (1972).
18. Kupčík F.: *Thesis*. Research Institute of Macromolecular Chemistry, Brno 1969.
19. Zwietering T. N.: Chem. Eng. Sci. *8*, 244 (1958).
20. Souza A. D., Pike R. W.: Can. J. Chem. Eng. *50*, 15 (1972).
21. Abramovich G. N.: *Teorija Turbulentnych Struj*. Gos. Izd. Fiz. Mat. Lit., Moscow 1960.
22. Donaldson C. D., Snedeker R. S.: J. Fluid Mech. *45*, 281 (1971).
23. Donaldson C. D., Snedeker R. S., Margolis D. P.: J. Fluid Mech. *45*, 477 (1971).
24. Knudsen J. G., Katz D. L.: *Fluid Dynamics and Heat Transfer*. McGraw-Hill, New York 1958.
25. Hrubý M.: Paper presented at the IV-th International CHISA Congress, Prague 1972.
26. Fořt I., Valešová M., Kudrna V.: This Journal *36*, 164 (1971).
27. Kupčík F.: Chem. průmysl 15/*40*, 415 (1965).

Translated by M. Rylek.